Theory of Spontaneous Emission Noise in Open Resonators and its Application to Lasers and Optical Amplifiers

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Abstract—A theory of spontaneous emission noise is presented based on classical electromagnetic theory. Unlike conventional theories of laser noise, this presentation is valid for open resonators. A local Langevin force is added to the wave equation to account for spontaneous emission. A general expression is found relating the diffusion coefficient of this force to the imaginary part of the dielectric function. The fields of lasers and amplifiers are found by solving the wave equation by the Green's function method. The lasing mode is a resonant state associated with a pole in Green's function. In this way, noise in lasers and amplifiers is treated by a unified approach that is valid for either gain guiding or index guiding. The Langevin rate equations for the laser are derived. The theory is illustrated with applications to traveling wave and Fabry-Perot amplifiers and Fabry-Perot lasers. Several new results are found: optical amplifier noise increases inversely with quantum efficiency; spontaneous emission into the lasing mode is enhanced in lasers with low facet reflectivities; and the linewidth of a Fabry-Perot laser with a passive section decreases as the square of the fraction of the cavity optical length that is active.

I. Introduction

THE TREATMENT of noise in semiconductor lasers has been quite successful in describing many properties including the frequency spectrum and amplitude distribution of intensity noise, the linewidth and line shape of single-mode lasers and mode partition noise. Despite these successes, the current theories are based on assumptions that are only approximately valid for most semiconductor lasers and amplifiers. The traditional theories of noise in lasers [1]–[3], begin by assuming that the laser can be treated as a *closed* cavity having a discrete set of orthogonal modes. The electromagnetic field is expanded in terms of these modes. In modeling the laser, facet losses are replaced by losses uniformly distributed throughout the cavity. This limitation of conventional laser noise theory is well known and has been addressed in a number of papers dealing with output coupling [4]–[6]. Neglect of facet losses is reasonable in the modeling of gas lasers with high reflecting facets, for which the theories of laser noise were originally developed. This approximation becomes more questionable when it is applied to a conventional semiconductor laser having cleaved facets with only 30-percent power reflectivities.

The expansion of the electromagnetic field in terms of discrete resonator modes cannot be applied to traveling wave optical amplifiers, because, ideally, these devices have no modes at all. In treating traveling wave and resonant optical amplifiers, a different approach is used [7]–[9]. The electromagnetic energy in each transverse mode mode is considered to be locally excited and then amplified during subsequent propagation in the device.

Another well-known example of an open cavity, where conventional laser noise theory fails, is the gain-guided laser. This problem was solved by Petermann [10], who showed that gain guiding can greatly enhance spontaneous into the fundamental transverse mode of a waveguide. Streifer *et al.* [11] applied Petermann's result to explain the multilongitudinal mode behavior of gain-guided lasers. However, Petermann did not derive his result from a general theory of laser noise and for this reason he stimulated many other papers, both defending and attacking him [12].[1] A general treatment of traveling wave amplifier noise, incorporating Petermann's result, has been given by Haus and Kawakami [13].

The purpose of this paper is to give a general formulation of spontaneous emission noise that is valid for open cavities and that can be readily applied to most semiconductor lasers and amplifiers. We will apply this theory to four cases: a traveling wave amplifier, a Fabry-Perot amplifier, a Fabry-Perot laser with arbitrary facet reflectivities, and the calculation of the Lorentz linewidth of a Fabry-Perot laser with a passive section. The theory is presented in a form suitable for application of other devices of current interest such as distributed feedback and coupled cavity lasers.

Our starting point is the same as that of Landau and Lifshitz, who treat electromagnetic fluctuations in dielectric media [14]. We assume that a field source (Langevin force) $F_\omega(x)$ accounts for spontaneous emission by the carriers at angular frequency ω. We find the mean square (diffusion coefficient) of the Langevin force, which is proportional to the imaginary part of the dielectric function $\epsilon_\omega(x)$.

When this source of spontaneous emission is included, the wave equation becomes inhomogeneous. The fields generated by this source may be calculated by the Green's function method. The Green's function depends on device

Manuscript received October 25, 1985.
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IEEE Log Number 8406984.

[1] M. Newstein, in [12], has criticized Petermann [10]. His criticism has to do with how spontaneous emission into the lasing mode is defined. The definition of spontaneous emission rate R used here in (48) leads to no disagreement with Newstein.

geometry and relates the universal noise source to the performance of specific devices.

Our treatment of laser noise is semiclassical. The quantum theory of radiation is not used and the amplitude of the noise source is determined merely by requiring that the spontaneous emission generated be exactly that required by statistical mechanics in the idealized case of an optical waveguide in equilibrium with a semiconductor. This approach is close to one we used earlier to relate the coefficients of absorption, gain, and spontaneous emission [15].

These methods are well known in electromagnetic theory, but have not been used very much in the description of lasers. Petermann's [7] calculation is essentially the calculation of a Green's function for a uniform waveguide. Our source of spontaneous emission resembles that of Haus and Kawakami [13], but we do not use an effective temperature as they do.

II. Spontaneous Emission Noise

A. Basic Equations

Our starting point is the wave equation for the electromagnetic field. For simplicity, we will avoid the use of vector fields. We assume that in the laser or amplifier, waves propagate primarily along the z axis. $E(\boldsymbol{x}, t)$ is the transverse electric field component with polarization vector in the x–y plane. If we describe the real electric field in terms of its complex Fourier components $E_\omega(\boldsymbol{x})$

$$E(\boldsymbol{x}, t) = \int_0^\infty E_\omega(\boldsymbol{x}) \exp(-i\omega t)\, d\omega + \text{c.c.} \tag{1}$$

where c.c. is the complex conjugate, the wave equation for $E_\omega(\boldsymbol{x})$ is

$$\nabla^2 E_\omega(\boldsymbol{x}) + \frac{\omega^2}{c^2} D_\omega(\boldsymbol{x}) = 0. \tag{2}$$

The electric displacement vector D_ω is assumed to be

$$D_\omega(\boldsymbol{x}) = \epsilon_\omega E_\omega(\boldsymbol{x}) + K_\omega(\boldsymbol{x}) \tag{3}$$

where the first term is due to the displacement induced by the electric field and the second term is a Langevin force associated with spontaneous polarizations set up by the semiconductor carriers [14]. The average value of K_ω is zero.

Substitution of (3) into (2) gives

$$\left[\nabla^2 + \frac{\omega^2}{c^2} \epsilon_\omega(\boldsymbol{x})\right] E_\omega(\boldsymbol{x}) = F_\omega(\boldsymbol{x}) \tag{4}$$

where Langevin force $F_\omega(\boldsymbol{x})$ is defined as

$$F_\omega(\boldsymbol{x}) \equiv -\frac{\omega^2}{c^2} K_\omega(\boldsymbol{x}). \tag{5}$$

B. Green's Function Solution of the Inhomogeneous Wave Equation

Equation (4) may be solved by first finding the Green's function $G_\omega(\boldsymbol{x}, \boldsymbol{x}')$ defined by

$$\left(\nabla^2 + \frac{\omega^2}{c^2} \epsilon_\omega\right) G_\omega(\boldsymbol{x}, \boldsymbol{x}') = \delta(\boldsymbol{x} - \boldsymbol{x}') \tag{6}$$

where $\delta(\boldsymbol{x} - \boldsymbol{x}')$ is the Dirac delta function. Then the field generated by spontaneous emission (and amplification) is given by

$$E_\omega(\boldsymbol{x}) = \int d\boldsymbol{x}'\, G_\omega(\boldsymbol{x}, \boldsymbol{x}')\, F_\omega(\boldsymbol{x}'). \tag{7}$$

For brevity, we use $\boldsymbol{x} = (x, z)$, where x represents both lateral coordinates x and y. We will assume that there is a complete set of transverse modes $\phi_n(x)$ satisfying

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\omega^2}{c^2} \epsilon_\omega(x)\right] \phi_n(x) = k_n^2 \phi_n(x). \tag{8}$$

The completeness relation is

$$\sum_n \frac{\phi_n(x)\, \phi_n(x')}{\langle \phi_n \phi_n \rangle} = \delta(x - x') \tag{9}$$

where

$$\int \phi_n \phi_m\, dx \equiv \langle \phi_n \phi_m \rangle = \delta_{nm} \langle \phi_n \phi_n \rangle.$$

In the case of index guiding, $\phi_n(x)$ is real and $\langle \phi_n \phi_n \rangle = 1$.

Making use of the completeness relation, we can expand the Green's function in terms of ϕ_n

$$G_\omega(\boldsymbol{x}, \boldsymbol{x}') = \sum_n g_n(z, z') \frac{\phi_n(x)\, \phi_n(x')}{\langle \phi_n \phi_n \rangle}. \tag{10}$$

The one dimensional Green's function $g_n(z, z')$ satisfies

$$\left[\frac{d^2}{dz^2} + k_n^2\right] g_n(z, z') = \delta(z - z') \tag{11}$$

as can be seen by substituting (9) and (10) into (6) and using (8). In some applications, such as the distributed feedback laser, k_n will also depend on z. The solution of (11) has been discussed at length by Morse and Feshback [16]. It is given by a one-dimensional Green's function

$$g_n(z, z') = \frac{Z_{n+}(z_>)\, Z_{n-}(z_<)}{W_n} \tag{12}$$

where $z_>$ and $z_<$ are the greater and lesser values of z and z' and $Z_{n+}(z)$ and $Z_{n-}(z)$ are the solutions of the homogeneous part of (11), satisfying the boundary conditions for positive and negative z, respectively. The Wronskian W_n is defined by

$$W_n \equiv Z'_{n+} Z_{n-} - Z_{n+} Z_{n-} \tag{13}$$

where $Z'_n \equiv dZ_n/dz$. It can be shown [15] that W_n is independent of z. the function $g_n(z, z')$ has a slope discontinuity allowing it to satisfy (11).

C. Laser Amplifier

We will be primarily interested in fundamental single transverse mode operation of amplifiers and lasers. In this case, we need only retain the $n = 0$ term in (10). For an amplifier extending from $z = 0$ to $z = L$, the field at L is given by

$$E_\omega(x, L) = Z_{0+}(L)\, \phi_0(x) \frac{\langle Z_{0-} \phi_0 F_\omega \rangle}{W_0 \langle \phi_0 \phi_0 \rangle} \tag{14}$$

where

$$\langle Z_{0-} \phi_0 F_\omega \rangle = \int_0^L Z_{0-}\, dz \int dx\, \phi_0(x)\, F_\omega(x, z).$$

D. Laser

In the case of a laser operating in the fundamental transverse mode, it is useful to think of W_0 as a function of the real angular frequency ω, which can be analytically continued to complex values $\tilde{\omega}$. The zero's of $W_0(\tilde{\omega})$ are the complex values $\tilde{\omega}_l$. These values $\tilde{\omega}_l$ are the poles of $E_\omega(x)$ in (14), which correspond to the longitudinal modes of the system. We see from (13) that at $W_0(\tilde{\omega}_l) = 0$, the functions $Z_{0+}(z)$ and $Z_{0-}(z)$ are proportional to one another. We will limit our discussion to single-mode lasing operation, i.e., we will assume that there is only a single pole $\tilde{\omega}_0$ near the real axis. We can define the functions Z_{0+} and Z_{0-} to be equal at this pole: $Z_{0+}(z) = Z_{0-}(z) \equiv Z_0(z)$ for ω near $\tilde{\omega}_0$. For a laser near threshold and for frequencies near $\tilde{\omega}_0$, we can expand W_0 as $W_0(\omega) = (dW_0/d\tilde{\omega})\,(\omega - \tilde{\omega}_0)$. The field (14) then becomes

$$E_\omega(x) = \frac{Z_0(z)\, \phi_0(x)\, \langle Z_0 \phi_0 F_\omega \rangle}{\langle \phi_0 \phi_0 \rangle \dfrac{dW_0}{d\tilde{\omega}} (\omega - \tilde{\omega}_0)}. \tag{15}$$

As the carrier number in the laser increases and threshold is approached both the real and imaginary parts of $\tilde{\omega}_0$ change. This is illustrated in Fig. 1. The pole $\tilde{\omega}_0$ can be expressed as

$$\tilde{\omega}_0 = \omega_0 + \frac{\alpha \Delta G}{2} + \frac{i \Delta G}{2} = \omega_0 + \frac{i \Delta G}{2}(1 - i\alpha) \tag{16}$$

where ω_0 is the cavity resonance at threshold, ΔG is the net gain (s^{-1}) and α is the linewidth parameter [17], which determines the rate of change of the cavity resonance with gain. The gain ΔG determines the rate of growth or decay of the field intensity in the absence of spontaneous emission. Below threshold, ΔG is negative. In a laser that is uniform along z, α is the ratio of the changes in the real and imaginary parts of the refractive index with carrier number [17]. Henry and Kazarinov [18] have used complex resonance frequency (16) to discuss mode suppression in cleaved coupled cavity lasers.

In a laser below threshold, ΔG, which is related to the carrier number N, can be taken to be a fixed parameter. Fluctuations in N can be neglected. Above threshold, N is affected by light intensity and the laser field is very sensitive to changes in N. All phenomena that distinguish the laser oscillator from a resonant amplifier, e.g., gain saturation (Fermi-level pinning); the change in the light intensity distribution from exponential to Gaussian; relaxation oscillations; and the enhancement of laser linewidth are due to the dynamic interaction of the carrier number and the light intensity. This interaction is most conveniently described by a set of coupled differential equations

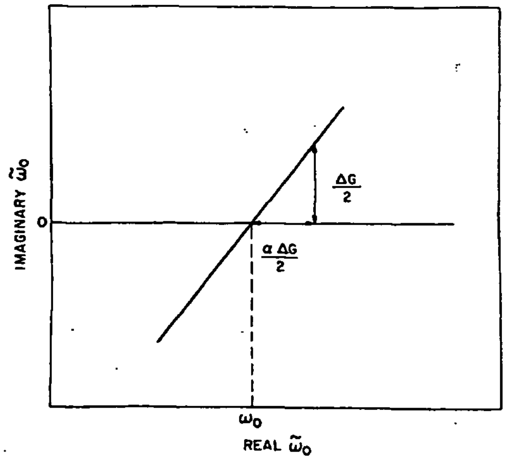

Fig. 1. The pole associated with the lasing mode in the complex $\tilde{\omega}$ plane.

for field amplitude $\beta(t)$ and carrier number N. The complex equation for $\beta(t)$ is often transformed into two real equations for light intensity $I(t)$ and phase $\phi(t)$.

We will now derive the equation for the complex field amplitude $\beta(t)$. We note that the spatial dependence of $E_\omega(x)$ in (15) is given by $Z_0(z)\, \phi_0(x)$ and define a field amplitude β_ω as

$$E_\omega(x) = B \beta_\omega Z_0(z)\, \phi_0(x). \tag{17}$$

Using (1) and (17), we can write the time dependent electric field $E(x, t)$ of the laser as

$$E(x, t) = B \beta(t)\, Z_0(z)\, \phi_0(x) + \text{c.c.} \tag{18}$$

where the field amplitude $\beta(t)$ is given by

$$\beta(t) = \int_0^\infty \beta_\omega \exp(-i\omega t)\, d\omega. \tag{19}$$

The constant B in (17) is for normalization. We choose its value (in Section V-B) so that the instantaneous number of photons in the cavity $I(t)$ (integrated electromagnetic energy divided by $\hbar\omega_0$) is given by

$$I(t) = \beta(t)\, \beta(t)^*. \tag{20}$$

An equation for $\beta(t)$ can be determined by multiplying (15) by $-i \int_0^\infty (\omega - \tilde{\omega}_0) \exp(-i\omega t)\, d\omega$ and using (16), (17), and (19). The result is

$$\dot{\beta} = \left[-i\omega_0 + \frac{\Delta G}{2}(1 - i\alpha) \right] \beta + F_\beta(t) \tag{21}$$

where

$$F_\beta(t) = \frac{-i \langle Z_0 \phi_0 F(x, t) \rangle}{B \langle \phi_0 \phi_0 \rangle \dfrac{dW}{d\tilde{\omega}}} \tag{22}$$

and

$$F(x, t) = \int_0^\infty F_\omega(x) \exp(-i\omega t)\, d\omega. \tag{23}$$

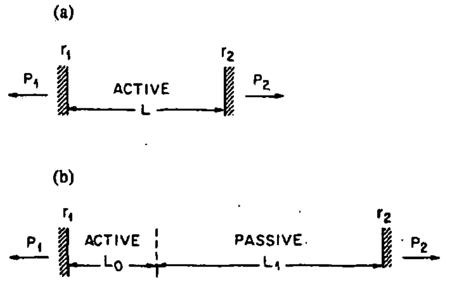

Fig. 2. Diagram defining the Fabry–Perot cavities considered in this paper: (a) uniform cavity, (b) cavity with passive section.

E. Green's Function for an Infinite Waveguide

We will now give several examples of Z_{n+1}, Z_{n-} and W_n that are useful later in this paper. For an unending waveguide (or a traveling wave amplifier with perfect antireflection AR coatings), we can choose

$$Z_{n\pm} = \exp(\pm ik_n z) \tag{24}$$

then the Wronskian (13) is

$$W_n = 2ik_n \tag{25}$$

and the one-dimensional Green's function (12) is

$$g_n(z, z') = \frac{\exp[ik_n|z - z'|]}{2ik_n}. \tag{26}$$

The complete Green's function (10) is

$$G_\omega(x, x') = \sum_n \frac{\phi_n(x)\,\phi_n(x')\exp[ik_n|z - z'|]}{2ik_n\langle\phi_n\phi_n\rangle}. \tag{27}$$

F. Green's Function for the Fabry–Perot Cavity

Consider a cavity extending from $z = 0$ to $z = L$ with field reflectivities r_1 and $z = 0$ and r_2 at $z = L$ (Fig. 2(a)). It is easily shown that for the fundamental transverse mode

$$Z_{0-} = r_1\exp(ik_0 z) + \exp(-ik_0 z) \tag{28}$$

satisfies the boundary condition at $z = 0$ and

$$Z_{0+} = r_1[\exp(ik_0 z) + r_2\exp(2ik_0 L - ik_0 z)] \tag{29}$$

satisfies the boundary condition at $z = L$.

The factor r_1 is added to (29) so that at threshold, when $r_1 r_2 \exp(2ik_{0\text{th}}L) = 1$, $Z_{0+} = Z_{0-}$. The Wronskian (13) for the fundamental mode is given by

$$W_0 = 2ik_0 r_1[1 - r_1 r_2\exp(2ik_0 L)]. \tag{30}$$

Near threshold $k_0 = k_{0\text{th}} + \Delta k_0$ and

$$1 - r_1 r_2\exp(2ik_0 L) = 1 - \exp(2i\Delta k_0 L) \approx -2i\Delta k_0 L \tag{31}$$

where

$$\Delta k_0 = \frac{\omega - \omega_0}{v_{g0}} - \frac{i}{2}\Delta g(1 - i\alpha) \tag{32}$$

and v_{g0} is the group velocity. Substitution of (32) with $\Delta G = v_{g0}\Delta g_0$ into (30) yields

$$W_0 = \frac{4k_0 L r_1}{v_{g0}}\left[\omega - \omega_0 - i\frac{\Delta G}{2}(1 - i\alpha)\right]. \tag{33}$$

Therefore, for the Fabry–Perot cavity

$$\frac{dW_0}{d\tilde{\omega}} = \frac{4k_0 L r_1}{v_{g0}}. \tag{34}$$

III. Diffusion Coefficients of the Langevin Forces

A. Diffusion Coefficients of $F_\omega(x)$

It can be generally established [19, sec. 118] from the requirement of stationarity, (i.e., that $\langle F(x, t)\,F(x', t + \tau)\rangle$ is independent of t), that

$$\langle F_\omega(x)\,F_{\omega'}(x')\rangle = \langle F_\omega(x)^*\,F_{\omega'}(x')^*\rangle = 0 \tag{35}$$

and that $\langle F_\omega(x)\,F_{\omega'}(x')^*\rangle$ is real and only nonzero when $\omega' = \omega$. We assume that the Langevin forces have negligible spatial correlation. (They should only be correlated over the distance a carrier can travel before being scattered, $\approx 10^{-2}$ μm). Therefore

$$\langle F_\omega(x)\,F_{\omega'}(x')^*\rangle = 2D_{FF^*}(x)\,\delta(x - x')\,\delta(\omega - \omega') \tag{36}$$

where the "diffusion coefficient" $D_{FF^*}(x)$ is real.

We can determine $D_{FF^*}(x)$ by requiring light traveling in a waveguide many absorption lengths long and having only bandgap absorption losses comes to equilibrium with the semiconductor. In equilibrium, the average noise power P_N in the z direction, within angular frequency interval $\Delta\omega$ will be $\Sigma_n\, v_{gn}\hbar\omega\langle n_\omega\rangle$ multiplied by the number of modes per unit length $(dk_n/d\omega)(\Delta\omega/2\pi)$, hence

$$P_N = \frac{\Delta\omega\hbar\omega\langle n_\omega\rangle}{2\pi}\sum_n \tag{37}$$

where $v_{gn} = d\omega/dk_n$ is the group velocity of the nth mode, Δk_n is the change in wave vector for change $\Delta\omega$, and the sum is over the transverse modes. In equilibrium, the chemical potential of the photons will be eV, the separation of the quasi-levels of the conduction band and valence band of the semiconductor. Then the change in free energy during emission or absorption is zero [15]. With this chemical potential, the average mode occupation number [19, sec. 53] is

$$\langle n_\omega\rangle = \frac{1}{\exp\left(\dfrac{\hbar\omega - eV}{kT}\right) - 1} \equiv -n_{sp}. \tag{38}$$

In equilibrium, $\hbar\omega > eV$ and $\langle n_\omega\rangle$ is positive. In a laser $\hbar\omega_0 < eV$, $\langle n_\omega\rangle$ is negative and the positive quantity n_{sp} is normally used [16].

We will now derive an expression for P_N (37) by use of the Green's function. Comparison of the two expressions for P_N will allow us to determine the diffusion coefficient $D_{FF^*}(x)$ (36). The power P_N is given by an integral over the z component of the Poynting vector $S = cE \times H/4\pi$

$$P_N = \frac{c\Delta\omega}{4\pi} \int dx \int_0^\infty d\omega' \langle (E_\omega H_{\omega'}^* \cdot \exp(-i(\omega - \omega')t) + \text{c.c.}) \rangle. \tag{39}$$

From Maxwell's equation

$$H_\omega = \frac{-ic}{\omega} \frac{\partial E_\omega}{\partial z}. \tag{40}$$

We will assume that our ideal waveguide is index guided so that ϕ_n is real and $\langle \phi_n \phi_n \rangle = 1$. Writing E_ω as a Green's function (7), (10), (12), and using (36), we find

$$P_N = \frac{c^2\Delta\omega}{4\pi\omega} \sum_n \frac{\langle \phi_n Z D_{FF^*} \phi_n \rangle}{|W_n|^2} \cdot (iZ_{n+}(L)\, Z_{n+}(L)'^* + \text{c.c.}) \int_0^L |Z_{n-}|^2\, dz. \tag{41}$$

Evaluating $Z_{n\pm}$ and W_n with (24) and (25), we find

$$P_N = \frac{c^2\Delta\omega}{16\pi\omega} \sum_n \frac{\langle \phi_n 2 D_{FF}^* \phi_n \rangle}{k_n' k_n''} \tag{42}$$

where $k_n = k_n' + ik_n''$. We have neglected $k_n''^2$ compared to $k_n'^2$ and approximated $1 - \exp(-2k_n'' L) \approx 1$. The later approximation is valid for a waveguide assumed to be many absorption lengths long. Using the equation for the lateral field (8) it is readily shown that

$$2k_n' k_n'' = \frac{\omega^2}{c^2} \langle \phi_n \epsilon'' \phi_n \rangle \tag{43}$$

where ϵ'' is the imaginary part of the dielectric constant ϵ. Therefore

$$P_N = \frac{\Delta\omega}{2\pi} \frac{c^4}{4\omega^3} \sum_n \frac{\langle \phi_n 2 D_{FF^*} \phi_n \rangle}{\langle \phi_n \epsilon'' \phi_n \rangle}. \tag{44}$$

This can be made to agree with the statistical mechanical result (37) if

$$2 D_{FF}^*(x) = \frac{4\omega^4 \hbar \epsilon''(x)}{c^4} \langle n_\omega \rangle. \tag{45}$$

A corresponding expression was derived by Landau and Lifshitz using quantum mechanics for thermal fluctuations of the electromagnetic field [14, eq. (88.11)]. Our expressions agree if we set the bias voltage $V = 0$ and add a term for the zero-point oscillations of the field by changing $\langle n_\omega \rangle$ to $\langle n_\omega \rangle + \frac{1}{2}$. Equation (45) is a form of the fluctuation dissipation theorem [18, sec. 123]. It expresses the fact that in equilibrium dissipation by optical absorption must be balanced by spontaneous emission causing field fluctuations. Having determined $2D_{FF^*}(x)$, we can then apply it to lasers and amplifiers which are generally not in equilibrium. Expressing $(\omega/c)\, \epsilon''(x) = -n'(x)\, g(x)$, where n' is the real part of the refractive index and $g(x)$ is the gain, (45) can be written as

$$2D_{FF^*}(x) = \frac{4\omega^3 \hbar}{c^3} n'(x)\, g(x)\, n_{sp} \tag{46}$$

where n_{sp} is the spontaneous emission factor (38).

B. Diffusion Coefficient for $F_\beta(t)$

The Langevin force $F_\beta(t)$ acting as a source of spontaneous emission into the lasing mode at ω_0 is defined by (22) and (23). Using these equations, $\langle F_\beta(t)\, F_\beta^*(t') \rangle$ is given by

$$\langle F_\beta(t)\, F_\beta(t')^* \rangle = \frac{\int_0^\infty d\omega \exp(-i\omega(t - t') \langle Z_0 \phi_0 2 D_{FF^*} Z_0^* \phi_0^* \rangle}{B^2 |\langle \phi_0 \phi_0 \rangle|^2 \left|\frac{dW_0}{d\bar{\omega}}\right|^2} \tag{47}$$

where the angular bracket on the right side is an integral over x and z. We are only interested in the frequency components of ω near ω_0. Therefore, we will approximate $2D_{FF^*}$ by its value at ω_0 and remove it from the integral. The remaining integral is $2\pi\delta(t - t')$. This approximation only amounts to replacing a narrow function by a delta function. Therefore

$$\langle F_\beta(t)\, F_\beta(t')^* \rangle = R\delta(t - t') \tag{48}$$

where

$$R = \frac{8\pi\omega_0^3 \hbar \langle \phi_0^* \phi_0 \rangle \langle Z_0^* n_0' g_0 n_{sp} Z_0 \rangle}{c^3 B^2 |\langle \phi_0 \phi_0 \rangle|^2 \left|\frac{dW_0}{d\bar{\omega}}\right|^2}. \tag{49}$$

We will show later that R is the average rate of spontaneous emission into the mode. A similar calculation, making use of (35), shows that

$$\langle F_\beta(t)\, F_\beta(t') \rangle = \langle F_\beta(t)^*\, F_\beta(t')^* \rangle = 0. \tag{50}$$

To complete the evaluation of R, the normalization constant B must be chosen. We choose B so that when the field is expressed in terms of β, the number of photons in the cavity is $I = |\beta|^2$. With this choise, the field β is the classical average of the photon annihilation operator b [20]. This choice is convenient because it allows the Langevin equations for light intensity I and carrier number N and the related diffusion coefficients to be expressed simply in terms of parameter R. In a dispersive medium, the electromagnetic energy per unit volume U is given by [14, eq. (61.10)]

$$U = \frac{1}{8\pi} \left[\frac{d}{d\omega} (\epsilon'\omega)\, E^2 + H^2 \right]. \tag{51}$$

Using (18) and (40), this becomes

$$U = \frac{C^2 \phi_0 \phi_0^*}{4\pi} \left[\frac{d}{d\omega} (\omega n'^2)\, |Z_0|^2 + \frac{c^2}{\omega^2} |Z_0'|^2 \right] |\beta|^2. \tag{52}$$

At any point in the cavity, Z consists essentially of backward and forward propagating waves with propagation constant $\omega n'/c$. In this case

$$\frac{c^2}{\omega^2} |Z_0'|^2 \approx n_0'^2 |Z_0|^2 \tag{53}$$

where only oscillatory terms, which contribute negligibly to the integral over the cavity length, have been neglected. Combining (52) and (53), we find

$$U = \frac{B^2 \phi_0 \phi_0^* |Z_0|^2 n_0' n_g}{2\pi} |\beta|^2 \tag{54}$$

where $n_g = \omega(dn'/d\omega) + n'$ is the group index. Equating $\int U dx/\hbar\omega_0$ to $|\beta|^2$, we find

$$B^2 = \frac{2\pi\hbar\omega_0}{\langle \phi_0^* \phi_0 \rangle \langle Z_0^* n_0' n_{g0} Z_0 \rangle} \tag{55}$$

where the bracket indicates an integral and $\langle \phi_0 n' n_g \phi_0^* \rangle \equiv n_0' n_g \langle \phi_0 \phi_0^* \rangle$. Substitution in (49) results in

$$R = \frac{4K\omega_0^2 \langle Z_0^* g_0 n_0' n_{sp} Z_0 \rangle \langle Z_0^* n_0' n_{g0} Z_0 \rangle}{c^3 \left| \frac{dW_0}{d\bar{\omega}} \right|^2} \tag{56}$$

where K is the gain guiding enhancement factor of Petermann [10]

$$K = \frac{\langle \phi_0^* \phi_0 \rangle^2}{|\langle \phi_0 \phi_0 \rangle|^2}. \tag{57}$$

IV. Application to Amplifiers

A. Traveling Wave Amplifier (TWA)

We will calculate the noise power P_N generated by spontaneous emission in a TWA having perfect antireflection facet coatings, assuming propagation in the fundamental mode and index guiding. The steps of calculation are the same as those used for deriving (44) with only one transverse mode $n = 0$ included. An additional loss per unit length α_0 is assumed, so that $k_0'' = (-1/2)(g_0 - \alpha_0)$. The diffusion coefficient $D_{FF^*}(x)$ in (44) is given by (46). The resulting noise power P_N is given by

$$P_N = \frac{\Delta\omega\hbar\omega g_0 n_{sp}}{2\pi(g_0 - \alpha_0)} [\exp((g_0 - \alpha_0) L) - 1]. \tag{58}$$

The power amplification of the signal is

$$A = \exp[(g_0 - \alpha_0) L]. \tag{59}$$

In a laser having no leakage currents, the quantum efficiency is given by

$$\eta \equiv \frac{g_0 - \alpha_0}{g_0}. \tag{60}$$

It is the ratio of stimulated emission not dissipated to total stimulated emission. With these definitions we can write

$$P_N = \frac{\Delta\omega\hbar\omega n_{sp}}{2\pi\eta} (A - 1). \tag{61}$$

We see that two nonideality factors increase noise: n_{sp} and η. In good, long wavelength lasers $n_{sp} \approx 2$ and $\eta \approx 0.5$, so that these factors increase the noise by about $\times 4$.

Equation (61) is well known (See [7]–[9], [13] and [21, references].) However, as far as we know, previous derivations use $\eta = 1$, thereby underestimating the noise figure by about $\times 2$.

B. Fabry–Perot Amplifier

In practice, end reflections are unavoidable and every TWA is actually a Fabry–Perot amplifier (FPA). We will calculate the noise power of an FPA under the same assumptions as made for the TWA: fundamental transverse mode operation, an additional distributed loss α_0 and index guiding. The noise power is calculated exactly as in the case of a TWA, except that $Z_{0\pm}$ and W_0 for the Fabry–Perot cavity (28)–(30) are inserted into (41). For simplicity we will regard the two facet field reflectivities as real and positive. The only effect of complex reflectivities is to introduce phase shifts which slightly alter the effective cavity length L. The integral of $|Z_{0-}|^2$ from 0 to L is given by

$$\langle |Z_{0-}|^2 \rangle = [r_1^2 + \exp(-(g_0 - \alpha_0) L)] \cdot \frac{\exp[(g_0 - \alpha_0) L] - 1}{g_0 - \alpha_0} \tag{62}$$

and

$$iZ_{0+}(L) Z_{0+}(L)'^* + \text{c.c.} \approx r_1^2(1 - r_2^2) \cdot \exp[(g_0 - \alpha_0) L] \frac{2\omega n_0'}{c}. \tag{63}$$

Upon substitution into (41), we find

$$P_N = \frac{\Delta\omega\hbar\omega n_{sp}}{2\pi\eta} [\exp((g_0 - \alpha_0) L) - 1] \cdot \left\{ \frac{[r_1^2 \exp((g_0 - \alpha) L) + 1] (1 - r_2^2)}{|1 - r_1 r_2 \exp(2ik_0 L)|^2} \right\}. \tag{64}$$

This differs from P_N for the TWA (58) by an additional resonant term in brackets which goes to 1 as r_1 and r_2 go to 0. With $\eta = 1$, (64) agrees with the expression given by Yamamoto [22] based on the earlier work of Gordon [8]. It is easily shown that optical power is amplified in passing through a FPA by

$$A = \frac{(1 - r_1^2)(1 - r_2^2) \exp[(g_0 - \alpha_0) L]}{|1 - r_1 r_2 \exp(2ik_0 L)|^2}. \tag{65}$$

Therefore,

$$P_N = \frac{\Delta\omega\hbar\omega n_{sp}}{2\pi\eta} [1 - \exp(-(g_0 - \alpha_0) L)] \cdot \frac{[r_1^2 \exp((g_0 - \alpha_0) L) + 1)]}{1 - r_1^2} A. \tag{66}$$

Near threshold $r_1 r_2 \exp[(g_0 - \alpha) L] \approx 1$ and

$$P_N \approx \frac{\Delta\omega\hbar\omega n_{sp}(1 - r_1 r_2)(r_1 + r_2)}{2\pi\eta(1 - r_1^2) r_2} A. \tag{67}$$

C. Enhancement of Amplifier Noise by Gain Guiding

In the case of gain-guided fundamental-mode operation, we obtain an increase in noise power flux P_N by a

factor K (57) which is exactly the result obtained by Petermann [10]. This is easily seen by examining (14) for the field of the fundamental mode at the end of an amplifier. Calculation of the noise power from (14) and (39) involves calculating

$$P_N \sim \frac{\langle \phi_0 \phi_0^* \rangle \langle \phi_0 \epsilon'' \phi_0^* \rangle}{|\langle \phi_0 \phi_0 \rangle|^2} = -\frac{\langle \phi_0 \phi_0^* \rangle^2}{|\langle \phi_0 \phi_0 \rangle|^2} \frac{c}{\omega} n_0' g_0 \quad (68)$$

where the last equality is easily derived from (8). Thus P_N is proportional to K which goes to 1 in the case of index guiding.

D. The Distribution of E_ω Emerging From a Chain of Optical Amplifiers

In calculating error rates in optical communications, the statistical distribution of the optical field at the detector is needed. The field (14) associated with amplified spontaneous emission E_ω is the sum (an integral) of many contribution $F_\omega(x)$. According to the central limit theorem [23] the distribution of the sum of many statistically independent random variables with common distributions is a Gaussian. Therefore, the real and imaginary parts of E_ω have Gaussian distributions. By applying (35) it is easily seen that the real and imaginary parts of E_ω have identical (statically independent) Gaussian distributions. That is, if E_ω can be represented as a vector in the complex plane. The distribution of this vector is isotropic and along any direction (fixed phase angle) the distribution is the same Gaussian.

When passed through subsequent linear amplifiers (having no gain saturation) the distribution of E_ω will be unchanged except in magnitude, but each amplifier in the chain will generate additional fields. It is well known that the sum of several Gaussian random variables is also a Gaussian random variable. Therefore, E_ω^{out} at the end of the amplifier chain will also have an isotropic Gaussian distribution. The power reaching the detector will be proportional to $\langle |E_\omega^{\text{out}}|^2 \rangle$ and of course it will be just the sum of amplified powers generated by each amplifier. It is easily shown that for a chain of M amplifiers of amplification A, in which each amplification is followed by an attenuation A^{-1} so that the net amplification of the chain is unity that the noise power at the output is

$$P_N(\text{out}) = \frac{MP_N}{A}. \quad (69)$$

The above analysis is much simpler than that of Yamamoto [22], who calculated noise of an amplifier chain from the viewpoint that the optical signals and noise are collections of photons having shot noise. In this paper, we emphasize the wave viewpoint. An analysis of an amplifier chain, similar to that given here has been made by Henry [24].

V. Application to Lasers

A. Langevin Equations for I and ϕ

The Langevin force equation for a single mode laser was derived earlier (21). The nonzero diffusion coefficient of the Langevin force $F_\beta(t)$ is determined by a single real parameter R given by (56). The fields are often more conveniently expressed in terms of intensity $I = \beta\beta^*$ and phase ϕ

$$\beta = I^{1/2} \exp\left[-i(\omega_0 t + \phi)\right] \quad (70)$$

or

$$\phi = -\omega_0 t + \frac{i}{2}(\ln \beta - \ln \beta^*). \quad (71)$$

The Langevin equations and associated diffusion coefficients are completely determined by the transformation equations (20) and (71) which define I and ϕ in terms of β, β^*, and t. Lax has given convenient and general formulas for obtaining the transformed Langevin equations and diffusion coefficients [25]. Making this transformation, we find

$$\dot{\phi} = \frac{1}{2}\alpha\Delta G + F_\phi(t) \quad (72)$$

$$\dot{I} = \Delta G I + R + F_I(t) \quad (73)$$

$$2D_{II} = 2RI \quad (74)$$

$$2D_{\phi\phi} = \frac{R}{2I} \quad (75)$$

$$2D_{I\phi} = 0. \quad (76)$$

Equation (73) shows that the average rate of spontaneous emission into the mode is R.

B. Power Flow in a Laser

The power P flowing along the z direction in a laser is given by integrating the Poynting vector over x. Expressing E with (18), H with (40) and dropping β^2 and β^{*2} which oscillate at twice the optical frequency, we find

$$P = \frac{c^2 B^2}{4\pi\omega_0}(iZ_0 Z_0'^* + \text{c.c.})\, I(t)\, \langle \phi_0 \phi_0^* \rangle. \quad (77)$$

Eliminating B^2 with (55), we have

$$P = \frac{\hbar c^2 (iZ_0 Z_0^{*\prime} + \text{c.c.})}{2\langle Z_0^* n_0' n_{g0} Z_0 \rangle} I(t). \quad (78)$$

In an open cavity, the definition of I is ambiguous because it depends upon what we choose to be the cavity length. The minimum extent of the cavity would have to include all active regions. If we choose to include passive regions beyond this length, I and $\langle Z_0 n_0' n_{g0} Z_0^* \rangle$ will increase, however, physical quantities such as P involve the ratio of these two quantities and are unambiguous.

C. Fabry-Perot Cavity Laser

In lasers such as the Fabry-Perot cavity laser $g_0 n_0$, and v_{g0} are constant along the cavity length and R (56) reduces to

$$R = \frac{4Kk_0^2 g_0 n_{sp0} \langle |Z_0|^2 \rangle^2}{v_{g0} \left|\frac{dW_0}{d\bar{\omega}}\right|^2} \quad (79)$$

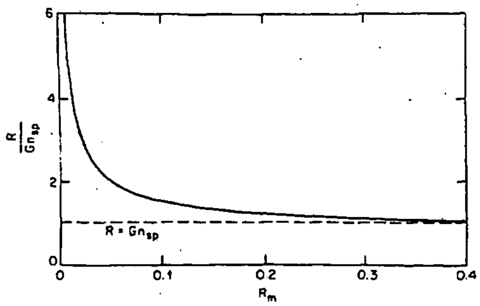

Fig. 3. Spontaneous emission rate R as a function of facet reflectivity.

where $k_0 = n_0' \omega_0 / c$. We can evaluate R for the Fabry–Perot laser with the aid of (34) and (62), we find

$$R = K\left[\frac{(r_1 + r_2)(1 - r_1 r_2)}{2 r_1 r_2 \ln (r_1 r_2)}\right]^2 G_0 n_{sp0} \qquad (80)$$

where r_1 and r_2 are the facet field reflectivities (Fig. 2(a)) and $G_0 = g_0 v_{g0}$. In the case of equal facet reflectivities $r_1 = r_2 = R_m^{1/2}$

$$R = K\left[\frac{(1 - R_m)^2}{R_m \ln (1/R_m))^2}\right] G_0 n_{sp0}. \qquad (81)$$

For index-guiding ($K = 1$) and high-reflecting facets ($r_1 = r_2 = 1$), $R = G_0 n_{sp0}$, which is the conventional result [17], [26]. For $R_m = 0.3$, corresponding to uncoated cleaved facets, the correction (bracketed term in (81)) is 1.13. For $R_m = 0.01$, the correction is 4.62. Hence the correction is modest for a conventional laser with uncoated cleaved facets and becomes quite significant for low-reflecting facets. The correction factor is plotted in Fig. 3.[2]

The facet powers of the Fabry-Perot laser can be easily found by evaluating P (78) at $z = 0$ and $z = L$ with the substitution of (28) and (62). At $z = 0$, $P_1 = -P$ is

$$\frac{P_1}{\hbar\omega_0} = v_{g0}\alpha_m I(t)\left[\frac{r_2(1 - r_1^2)}{(r_1 + r_2)(1 - r_1 r_2)}\right] \qquad (82)$$

and at $z = L$, $P_2 = P$ is

$$\frac{P_2}{\hbar\omega_0} = v_{g0}\alpha_m I(t)\left[\frac{r_1(1 - r_2^2)}{(r_1 + r_2)(1 - r_1 r_2)}\right] \qquad (83)$$

where $\alpha_m \equiv L^{-1} \ln ((r_1 r_2)^{-1})$. When $r_1 = r_2$, the bracketed terms reduce to $\frac{1}{2}$.

D. Linewidth of Fabry-Perot Laser Containing a Passive Section

The broad linewidth of semiconductor lasers can be reduced considerably by adding a long passive section to the laser cavity. We will calculate the linewidth reduction for an ideal Fabry-Perot cavity with a passive section shown in Fig. 2(b) in which there is a nonreflecting transition between the active and passive sections.

In this case, the Wronskian for the fundamental transverse mode is

$$W_0 = 2ik_0 r_1[1 - r_1 r_2 \exp (2ik_0 L_0 + 2ik_1 L_1)]. \qquad (84)$$

It agrees with (30) except for the additional term $2ik_1L_1$. At threshold $\omega = \omega_0$ and $r_1 r_2 \exp (2ik_0 L_0 + 2ik_1 L_1) = 1$. We can expand W_0 about ω_0 as before, see (30)–(34), but including an additional term $2\Delta k_1 L_1 = 2(\omega - \omega_0) L_1/v_{g1}$ associated with the passive region. Expanding $W_0(\omega)$ as $dW_0/d\tilde{\omega}\ (\omega - \omega_0 - i\Delta G(1 - i\alpha)/2)$, we find that compared with the values for a uniform Fabry–Perot cavity $(dW_0/d\tilde{\omega})$ increases by ξ^{-1} and ΔG decreases by ξ, where

$$\xi = \frac{L_0/v_{g0}}{L_0/v_{g0} + L_1/v_{g1}} \qquad (85)$$

is the fraction of the cavity optical length occupied by the active region.

The function $Z_0(z)$ (28) is unchanged by the addition of the passive section. If we define I as the number of photons in the active section, the spontaneous emission rate R (79) is unchanged except for a decrease by ξ^2 due to the change in $dW_0/d\tilde{\omega}$. Furthermore, with this definition of I, the relation of I and output power P_1 (82) is unchanged. The Lorentzian linewidth of the laser $\Delta\nu$ is given by the (spontaneous emission rate) $(1 + \alpha^2)/4\pi I$ [17], [26]. In this case, we have

$$\Delta\nu = \frac{R(1 + \alpha^2)\xi^2}{4\pi I} \qquad (86)$$

where R is given by (80), (81). Using (82) to relate I to the power out of the active end P_1, we have

$$\Delta\nu = \frac{\hbar\omega_0 g_0 n_{sp0} v_{g0}^2 \xi^2}{16\pi L_0 P_1} \cdot \left[\frac{(r_1 + r_2)(1 - r_1 r_2)(1 - r_1^2)}{r_1^2 r_2 \ln (r_1^{-1} r_2^{-1})}\right](1 + \alpha^2). \qquad (87)$$

For equal reflectivities $r_1 = r_2 = R_m^{1/2}$, the linewidth reduces to

$$\Delta\nu = \frac{\hbar\omega_0 g_0 n_{sp0} v_{g0}^2 \xi^2}{8\pi L_0 P_1}\left[\frac{(1 - R_m)^2}{R_m \ln (R_m^{-1})}\right](1 + \alpha^2). \qquad (88)$$

Compared with the Lorentzian linewidth calculated in [17], [26] for a uniform laser, $\Delta\nu$ is increased by the enhancement in spontaneous emission (81) and decreased by ξ^2. Line narrowing by a passive section can be substantial. For $L_0 = 300\ \mu$m, $v_{g0} = c/4$, $L_1 = 3$ cm, and $v_{g1} = c1.5$, we find $\xi = \frac{1}{39}$ and $\xi^2 = \frac{1}{1480}$. This value of ξ will reduce a linewidth of 100 MHz to 67 kHz.

Summary and Discussion

A theory of spontaneous emission noise in semiconductor lasers and amplifiers is presented. The presentation is self-contained and based on classical electromagnetic theory. A local source of spontaneous emission in the form of a Langevin force $F_\omega(x)$ is added to the wave equation. The diffusion coefficient for this noise source is found by requiring consistency with the results of equilibrium statistical mechanics. It is shown to be proportional to the

[2] This same correction factor was found by Ujihara [27] using a different approach.

imaginary part of the dielectric function $\epsilon''_\omega(x)$ and the spontaneous emission factor n_{sp} (38), determining the degree of inversion of the amplifying medium.

The Green's function method is used to relate the field $E_\omega(x)$ to the noise source. In this way a theory of spontaneous emission noise is developed that is valid for open resonators. It differs from conventional laser noise theory which relies upon expansion of the electromagnetic field in terms of the normal modes of a closed resonator [1]-[3].

The theory is applied to amplifiers and expressions are derived for the power spontaneously emitted from traveling wave and Fabry-Perot amplifiers. These expressions are conventional except that they contain an excess noise factor η^{-1}, where η is the quantum efficiency of the amplifier due to nonbandgap absorption and scattering losses. This factor appears to have been missed in earlier treatments of optical amplifiers. It is shown that the distributions real and imaginary parts of the field generated by amplified spontaneous emission are identical Gaussians and that this result is valid for a single amplifier or a chain of linear amplifiers and attenuators.

The lasing mode is shown to be associated with the pole of the Green's function. In this way, the same methods can be applied to treat lasers and amplifiers. The Langevin rate equations for the laser field, intensity and phase are derived. The various diffusion coefficients of the Langevin forces are shown to depend on a single parameter R, the spontaneous emission rate. A general formula for R is given (56). This formula is evaluated for the Fabry-Perot cavity. It is shown that for a Fabry-Perot cavity with high-reflecting ends, R reduces to the conventional value $G_0 n_{sp}$, where G_0 is the gain per second. For lower values of reflectivity corresponding to an open cavity, R is enhanced (Fig. 3).[3] A Fabry-Perot cavity laser with a passive section is also treated. It is shown that the linewidth of such a laser can be reduced by ξ^2 where ξ is given by the fraction of the optical length of the cavity occupied by the active region (85).

Our theory is valid for gain-guided as well as for index-guided lasers. We find that for gain-guided single-transverse mode lasers and amplifiers, spontaneous emission is enhanced by the K factor of Petermann [10].

In conclusion, we have presented a self-contained theory of laser noise, valid for open resonators, that unites the treatments of amplifiers and gain guiding with the conventional treatment of lasers.

NOMENCLATURE

$E(x, t)$	Electric field.
$E_\omega(x)$	Fourier component of electric field.
$D_\omega(x)$	Fourier component of electric displacement.
$K_\omega(x)$	Contribution to D_ω by spontaneous polarizations.
$F_\omega(x) = (-\omega^2/c^2)\, K_\omega(x)$	Langevin force of the wave equation (4).
$\epsilon_\omega(x)\ \epsilon'(x) + i\epsilon''(x)$	Dielectric function.
$G_\omega(x, x')$	Greens functions satisfying (6).
$\phi_n(x)$	nth transverse mode.
k_n	Propagation vector of nth transverse mode (8).
$g_n(z, z')$	Green's function for the z coordinate of mode n (11).
$Z_{n\pm}(z)$	Solutions of homogeneous part of (11).
W_n	Wronskian (12), (13).
$\tilde{\omega}_0$	Pole associated with the lasing mode (16).
ω_0	Threshold angular frequency of fundamental mode (16).
ΔG	Net gain (s^{-1}) of fundamental mode (16).
α	Linewidth parameter of fundamental mode (16).
$Z_0(z)$	z dependence of fundamental lasing mode (17), (18).
$\beta(t)$	Amplitude of the laser mode (18).
β_ω	Fourier component of $\beta(t)$ (19).
B	Normalization constant (17).
$I = \lvert\beta\rvert^2$	Intensity of lasing mode (20).
$F_\beta(t)$	Langevin force driving laser field amplitude (23), (24).
$dW_0/d\tilde{\omega}$	Derivative of W_0 used to expand W_0 near laser mode (15).
v_{g0}	Group velocity of the fundamental mode.
$D_{FF^*}(x)$	Diffusion coefficient of $F_\omega(x)$ (36).
$\langle n_\omega \rangle = -n_{sp}$	Equilibrium mode occupation number (38).
P_N	Noise power of amplifier (42).
$n'(x) + in''(x)$	Refractive index.
$g(x) = (-\omega/c)\, n(x)''$	Gain per centimeter.
n_0, g_0, α_0	Refractive index, gain, and loss of the fundamental mode.
$G_0 = g_0 v_{g0}$	Gains/s of fundamental mode.
R	Spontaneous emission rate (56).
A	Power amplification.
L	Length of Fabry-Perot cavity (Fig. 2(a)).

[3](After completing this paper it was found that Ujihara [27] had found the same effect.)

r_1, r_2	Field reflectivities of Fabry–Perot cavity (Fig. 2).
R_m	Facet power reflectivity for $r_1 = r_2$.
K	Enhancement of spontaneous emission by gain guiding (57).
η	Quantum efficiency (60).
ϕ	Phase of laser field (70).
D_{II}, $D_{\phi\phi}$, $D_{I\phi}$	Diffusion coefficients of intensity and phase for laser (74)–(76).
U	Electromagnetic energy density (51), (54).
ξ	Fraction of optical length occupied by active region ((85), Fig. 2(b)).
L_0, L_1	Active and passive lengths in Fig. 2(b).

Acknowledgment

The author wishes to thank N. A. Olsson, M. Lax, G. P. Agrawal, P. S. Henry, and M. Newstein for stimulating discussions.

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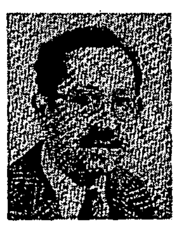

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